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ABSTRACT

An analysis of a reactor fuel pin with a central void is presented which considers stresses in the clad caused by fission gas pressure. It is shown that, for a given fuel volume, the clad and void volumes may be optimized to give a minimum stress in the clad. Stress and strain in the clad may be calculated from equations and graphs of a stress parameter as a function of a heat generation parameter and the fuel-to-pin volume ratio. An illustrative example is presented showing how the analysis and creep properties of the clad may be used in conjunction with reactor criticality calculations.

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SUMMARY

High fuel burnup yields a proportionately high quantity of fission products. Fission products may cause fuel swelling and a buildup of pressure within the fuel element. The case of fission gas release from the fuel is considered in which the effect of fuel swelling on the clad stress may be neglected. An analytical investigation has been made of a cylindrical fuel element for the hoop stress in the clad material resulting from a pressure buildup due to fission gas accumulation within a central void. The analysis shows that for a selected fuel volume and pin size the void and clad volumes may be optimized to give a minimum stress in the clad. Equations and graphs are presented for a grouping of terms showing a stress parameter as a function of a heat generation parameter and the fuel-to-pin volume ratio.

An example is presented where the graphs of this report and a reactor criticality curve were used to relate the clad stress to reactor diameter. The limiting stress is determined from creep properties of the clad.

INTRODUCTION

High power density nuclear reactors designed for long operating life generate a high concentration of fission products. Because fission products cause the fuel to swell if retained in the fuel, or generate pressure on the clad material if released from the fuel, some means must be provided to accommodate the fission products. Otherwise, the fuel elements may rupture or undergo significant dimensional changes as a result of creep. The latter is particularly important for high-temperature, long-life applications such as space power systems. Cylindrical fuel pins incorporating an appreciable void space has been considered to accommodate fuel swelling and fission gases, see for example, refs. 1 to 3. Reference 1 suggests further that the fuel be constructed so that

it is mechanically weak and that the clad be made strong. Reference 3 considers the potential advantages of vapor transport of the fuel when operating at high fuel temperatures.

This report also utilizes a cylindrical fuel pin with a void space and analyzes the effect of design variables and operating conditions on the stress in the fuel pin clad resulting from fission gas pressure buildup. Stresses in the clad which may be caused by fuel swelling are not included. It is recognized that such stresses are not always negligible. However, some designs and operating conditions can be envisioned where the stresses imposed by fuel swelling are small relative to the pressure stresses. For example, stress in the clad caused by fuel swelling will be small when the strength of the fuel is low relative to that of the clad or when conditions are such that most of the fission products are released from the fuel. In cases where such stresses are not negligible, the analysis obviously yields optimistic results. An analysis of the combined stresses and strains in the fuel and clad would then be required, which is beyond the scope of this report.

Fuel pin variables may effect the overall reactor design. For example, changing the fuel volume fraction in the pin would effect reactor criticality. Therefore, in order to coordinate the design of the fuel pin with the reactor, care must be taken in the fuel pin analysis to preclude undesirable effects on the entire core. To separate the fuel pin analysis from criticality considerations, arbitrary values of fuel-to-pin volume ratios were selected covering a range of 0.1 to 0.6. Thus, for each fuel-to-pin volume ratio selected, the void and clad volumes can be exchanged to achieve a minimum stress condition in the clad. This exchange of void and clad volumes has only a minor effect on the neutronics of the core in a fast fission spectrum reactor, because in these reactors the neutron absorption in the clad material is minimal. Thermal reactors, however, may require a compromise between stress and neutron absorption depending on the clad material. The analysis yields optimum combinations of void and clad volumes based on pressure stresses. The results are presented in terms of dimensionless parameters from which stresses and strains may be calculated.

ANALYSIS

A fuel element for a reactor with a high percentage of fuel burnup may be a cylindrical type element shown in figure 1. An analysis will be made on this model for the stress on the clad material resulting from a pressure acting on the inside surface of the clad. The fuel element consists of three volumes: the fuel volume, the central void volume where the fission gases collect, and the clad volume for the containment

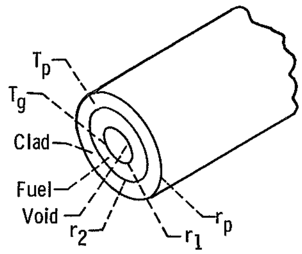


Figure 1. - Cylindrical fuel pin with central void.

of the fission gases. The state of the fission gases within the void volume may be described by the perfect gas law expressed in the form shown by equation (1).

$$pV_v = NKT_g \tag{1}$$

where N equals the number of molecules and K is the Boltzmann constant. All symbols are defined in the appendix. Assuming the gaseous fission products are monatomic (primarily Xe and Kr), the number of molecules (N) released into the void volume can be expressed by the following equation:

$$N = 2N_F C_Y C_R \tag{2}$$

where the number 2 is the ratio of number of fission products to the number of fissions, N_F is the number of fissions that occur during the life of the reactor, C_Y is the fraction of the fission products that are gaseous, and C_R is the fraction of fission gases that are released from the fuel into the original void volume.

Stresses in the clad caused by fuel swelling are assumed to be negligible in the case considered herein. Clad stresses are then those caused by fission gas pressure. For an internally pressurized thick-walled cylinder under steady-state creep, the tangential stress is a maximum at the outside radius and is given by the following equation from reference 4:

$$\sigma = \frac{2p}{n\left[\left(\frac{r_p}{r_2}\right)^{2/n} - 1\right]} \tag{3}$$

where r_p is the outside radius of the fuel pin clad, r_2 is the inside radius of the fuel pin clad, and n is the exponent on stress in the creep rate equation of the form $\dot{\epsilon} = a\sigma^n$. The temperature drop across the clad is assumed to be small such that the thermal

stresses and variation in creep properties can be neglected.

Elastic stress may be obtained from equation (3) by using a value of n equal to 1. For the case of n equal to 2, equation (3) reduces to

$$\sigma = \frac{p}{\frac{r_p}{r_2} - 1} \tag{3a}$$

which is equivalent to the simple thin-walled cylinder stress equation

$$\sigma = \frac{pr_2}{t} \tag{3b}$$

where t is the clad wall thickness. It will be convenient to relate stress for other values of n to that given by equation (3b). The ratio of the stress for any value of n to the stress at n equal to 2 is given by

$$\frac{\sigma}{\sigma_{n=2}} = \frac{2\left(\frac{r_p}{r_2} - 1\right)}{n\left[\left(\frac{r_p}{r_2}\right)^{2/n} - 1\right]} \tag{4}$$

The volumes comprising the fuel pin may be expressed as fractions so that

$$\frac{V_v}{V_p} + \frac{V_f}{V_p} + \frac{V_c}{V_p} \equiv 1 \tag{5}$$

The radius ratio r_p/r_2 in equation (3) may be expressed in terms of volume fractions so that

$$\left(\frac{r_p}{r_2}\right)^2 = \frac{V_p}{V_f + V_v} \tag{6}$$

Combining equations (1), (2), (3), and (6) the stress equation becomes:

$$\sigma = \frac{4C_YC_RKT_gN_F}{nV_v\left[\left(\frac{V_p}{V_f + V_v}\right)^{1/n} - 1\right]} \tag{7}$$

Letting f equal the number of fissions per unit fuel volume (N_F/V_f) and rearranging equation (7) yields a nondimensional stress parameter

$$B = \frac{\sigma}{C_YC_RKfT_p} = \frac{4T_g}{nT_p}\frac{V_f}{V_v}\frac{1}{\left[\left(\frac{V_p}{V_f + V_v}\right)^{1/n} - 1\right]} \tag{8}$$

The temperature ratio T_g/T_p can be obtained from the temperature profile in a hollow cylinder with internal heat generation. Assuming the heat source in the fuel to be uniform, the general equation for temperature with heat conduction from reference 5 is

$$T = -\frac{qr^2}{4k} + C_1 \ln r + C_2 \tag{9}$$

where q is the heat generation rate per unit volume of fuel, and k is the thermal conductivity of the fuel. The boundary conditions are (see fig. 1)

$$\frac{dT}{dr} = 0 \quad \text{at} \quad r = r_1$$

$$T = T_p \quad \text{at} \quad r = r_2$$

Using these boundary conditions in equation (9), the temperature ratio T_g/T_p may be written as

$$\frac{T_g}{T_p} = 1 + \frac{qr_2^2\left[1 - \left(\frac{r_1}{r_2}\right)^2 - 2\left(\frac{r_1}{r_2}\right)^2 \ln\frac{r_2}{r_1}\right]}{4kT_p} \tag{10}$$

where the fission product gas temperature is taken to be equal to the inside fuel temper-

ature. The temperature drop through the clad was assumed to be small and was therefore neglected.

The heat generation rate per unit fuel volume at constant reactor power is

$$q = \frac{Ef}{\tau} \tag{11}$$

where E is the heat energy per fission and τ is the operating time at power.

Expressing the radius ratios of equation (10) as volume ratios and combining with equation (11) yields

$$\frac{T_g}{T_p} = 1 + \frac{EfD_p^2}{16k\tau T_p}\left[\frac{V_f}{V_p} - \frac{V_v}{V_p}\ln\left(1 + \frac{V_f}{V_p}\frac{V_p}{V_v}\right)\right] \tag{12}$$

Defining a dimensionless heat generation parameter as

$$A = \frac{EfD_p^2}{16k\tau T_p} \tag{13}$$

and combining equations (8), (12), and (13) yields

$$B = \frac{\left(\frac{4}{n}\right)\left(\frac{V_f}{V_p}\right)\left(\frac{V_p}{V_v}\right)}{\left[\left(\frac{V_p}{V_f + V_v}\right)^{1/n} - 1\right]}\left\{1 + A\left[\frac{V_f}{V_p} - \frac{V_v}{V_p}\ln\left(1 + \frac{V_f}{V_p}\frac{V_p}{V_v}\right)\right]\right\} \tag{14}$$

The dimensionless stress parameter is a function of A, n, V_f/V_p, and V_v/V_p. Variations in V_f/V_p affect the stress parameter and therefore the stress in the clad but also affect the core neutronics. However, for any fixed value of V_f/V_p, V_v/V_p can be varied with only a minor effect on core neutronics. For example, the critical size of a given type reactor (fixed moderator and coolant volume fractions) is primarily a function of fuel volume fraction. (This is particularly true of a fast spectrum reactor where a variation in clad volume has a minimal effect on neutron absorption. For a thermal reactor the degree of freedom to vary the clad volume without a significant effect on the core neutronics is dependent on the clad material being used.) Thus, for any given fuel-to-pin volume ratio, the void and clad volumes in the fuel pin may be exchanged to achieve a minimum stress condition with only a minor effect on the entire

core. The effect of exchanging void and clad volumes is as follows. Increasing the void volume and decreasing the clad volume results in a decrease in fission gas pressure but ultimately leads to a finite pressure with zero clad thickness and an infinite stress. Decreasing the void volume and increasing the clad volume leads to an infinite pressure with a finite clad thickness and infinite stress. The condition for minimum stress can be found by differentiating equation (14) with respect to V_v/V_p for a constant value of A, n, and V_f/V_p, and setting the derivative equal to zero.

$$\frac{dB}{d\left(\frac{V_v}{V_p}\right)} = 0$$

$$= n\left(\frac{V_f}{V_p} + \frac{V_v}{V_p}\right)^{(n+1)/n} - n\left(\frac{V_f}{V_p}\right) - (n - 1)\left(\frac{V_v}{V_p}\right)$$

$$+ A\left[n\left(\frac{V_f}{V_p}\right)^2\left(\frac{V_f}{V_p} + \frac{V_v}{V_p}\right)^{1/n} - n\left(\frac{V_f}{V_p}\right)^2 + \left(\frac{V_f}{V_p}\right)\left(\frac{V_v}{V_p}\right) - \left(\frac{V_v}{V_p}\right)^2 \ln\left(1 + \frac{V_f}{V_v}\right)\right] \tag{15}$$

For the case of n equal to 2 (equivalent to the stress in a thin-walled cylinder, eq. (3b)) and A equal to zero, equation (15) in combination with equation (5) reduces to

$$\frac{V_f}{V_p} = 2\left(1 - \frac{V_c}{V_p}\right)^{3/2} - \left(1 - \frac{V_c}{V_p}\right) \tag{16}$$

The special case of A equal to zero corresponds to a temperature drop across the fuel of zero ($T_g = T_p$). This hypothetical situation may result from zero heat generation in the fuel or a fuel having infinite thermal conductivity. This special case will be used to optimize void and clad volumes for selected fuel volumes. Therefore, equation (16) sets the optimum relation between V_c/V_p and V_f/V_p to give a minimum value of the stress parameter B for the case of A equal to zero and n equal to 2. Since V_c/V_p is fixed by equation (5), an optimum void-to-clad volume ratio (V_v/V_c) exists for each value of V_f/V_p. All other values of V_v/V_c give a higher value of the stress parameter. The value of the stress parameter may be obtained from equation (14).

DISCUSSION

An optimum combination of void and clad volumes exists for each value of fuel-to-pin volume ratio. This combination yields a minimum stress in the clad as discussed in the ANALYSIS section. The void and clad volume ratios (V_v/V_p and V_c/V_p) which yield the minimum value of the stress parameter B were determined from equations (16) and (5), respectively, for the case of A equal to zero and n equal to 2. The results are shown in figure 2 as a function of fuel volume ratio V_f/V_p. Also shown in figure 2 are the clad thickness to diameter ratio t/D_p and the fuel-to-clad thickness ratio t_f/t.

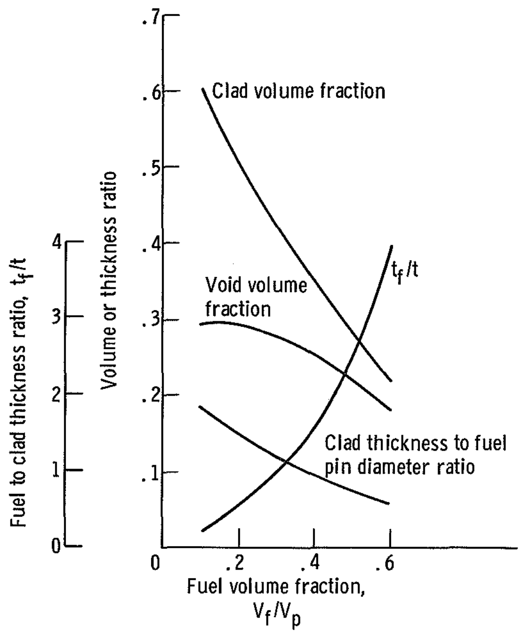
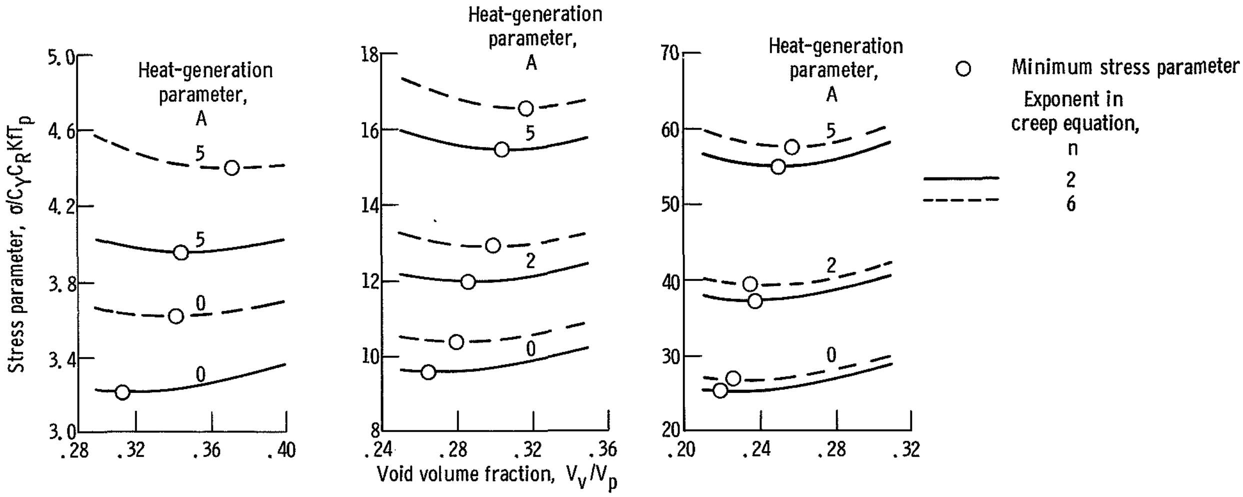


Figure 2. - Variation of volume or thickness ratio with respect to fuel volume fraction for minimum stress in clad. Exponent on stress in creep equation, n, 2; heat-generation parameter, A, 0.

At a V_f/V_p ratio of 0.6, the void volume and clad volume are nearly identical and equal to approximately 20 percent of the pin volume. At a V_f/V_p ratio of 0.2, the clad volume is considerably greater than the void volume, about 50 percent clad volume and 30 percent void volume. The t/D_p ratio varies from about 0.06 to 0.15. A 1-inch (25.4-mm) diameter pin and a fuel loading of 60 volume percent ($V_f/V_p = 0.6$) would require a clad thickness of 0.05 inch (1.27 mm). The required thickness for a V_f/V_p ratio of 0.2 is 0.15 inch (3.8 mm).

The variation in the stress parameter about the minimum is shown in figure 3. The



(a) Fuel volume fraction, 0.20.

(b) Fuel volume fraction, 0.35.

(c) Fuel volume fraction, 0.50.

Figure 3. - Variation of stress parameter about minimum.

stress parameter is plotted as a function of V_v/V_p as determined by equation (14) for selected values of A, n, and V_f/V_p. The circle symbols denote the minimum value determined from equation (15). The optimum value of V_v/V_p varies about 20 percent from A = 0, n = 2 to A = 5, n = 6 for each of the three values of V_f/V_p shown. The curves are seen to be flat in the region of the minimum. A small deviation from the optimum values of V_v/V_p results in only a slight increase in stress from its minimum value. The minimum stress parameter on the A = 5, n = 6 curve of figure 2(b) has a value of 16.6. The value of the stress parameter on the same curve is 17.0 for V_v/V_p of 0.265 (the optimum for the A = 0, n = 2 curve), or approximately $2\frac{1}{2}$ percent higher than the minimum. The same difference exists for V_f/V_p ratios of 0.20 and 0.50 (figs. 3(a) and (c)). Therefore, the proportioning of the void and clad volumes (V_v/V_p and V_c/V_p) may be obtained for an A value of zero and an n value of 2 with only a slight increase in the stress parameter. Proper values of A and n should, however, be used in determining the value of the stress parameter from equation (14).

The curves of figure 2 can therefore be used to approximate the optimum combination of V_c/V_p and V_v/V_p, regardless of the value of A and n.

Since small variations in the void and clad volumes from their optimum values do not produce a significant change in the stress parameter, adjustments may be made to minimize fuel swelling effects by increasing the clad thickness or to reduce parasitic neutron absorption by decreasing clad thickness.

The heat generation parameter was introduced in equation (13) and is a measure of the T_g/T_p ratio. When designing a reactor with a high fuel pin heat flux, the fission gas temperature or inner fuel temperature should be calculated. A high heat generation

rate and/or thick fuel could give a calculated fuel temperature above its melting temperature at the inner surface. A plot of T_g/T_p (eq. (12)) for a range of V_f/V_p and heat generation parameter for a minimum stress condition is shown in figure 4. A fuel pin with a large percent of fuel and a large heat generation parameter can easily have a T_g/T_p ratio of 2. One of the assumptions for determining the T_g/T_p ratio (eq. (12)) was that no thermal resistance exists between the fuel and clad. Some correction factor to account for this thermal resistance may be required when calculating the fission gas temperature in a high heat flux fuel element. This correction need not be considered when calculating the optimum proportioning of the fuel element (eq. (15)), only a slight error will be introduced.

The variation of the stress parameter with V_f/V_p for n equal to 2 and for several values of the heat generation parameter A is shown in figure 5.

The range of the heat generation parameter of zero to 5 and the range of V_f/V_p ratio of 0.1 to 0.6 were arbitrarily chosen, but should encompass most practical high-temperature conditions. A heat generation parameter A of zero is the limiting case of $T_g = T_p$. The optimum relation of the ratios V_f/V_p and V_v/V_p (eq. (16), fig. 2) is incorporated in the figure. A plot similar to figure 5 but for a family of lines for the V_f/V_p ratio is shown in figure 6. This latter figure is more usable for design because interpolation between curves is more nearly linear.

The value of the stress parameter for n equal to 2 can be obtained from figures 5

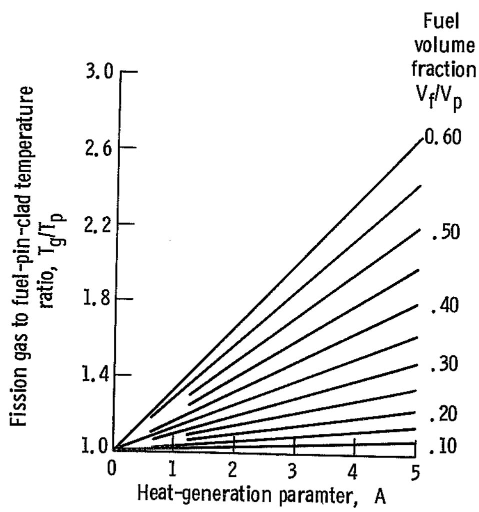


Figure 4. - Variation of fission gas to fuel-pin-clad temperature ratio with respect to heat-generation parameter and for several values of fuel to pin volume ratio.

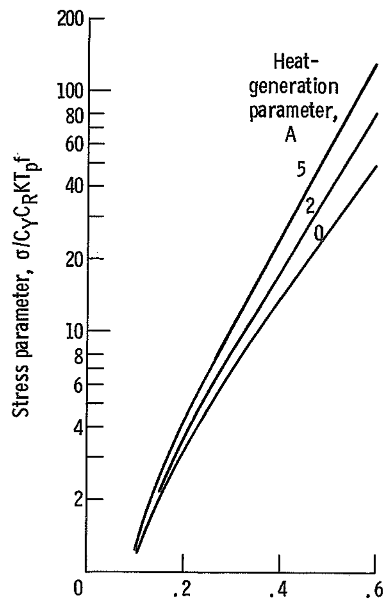


Figure 5. - Stress parameter variation with respect to fuel volume fraction. Exponent on stress in creep equation, 2.

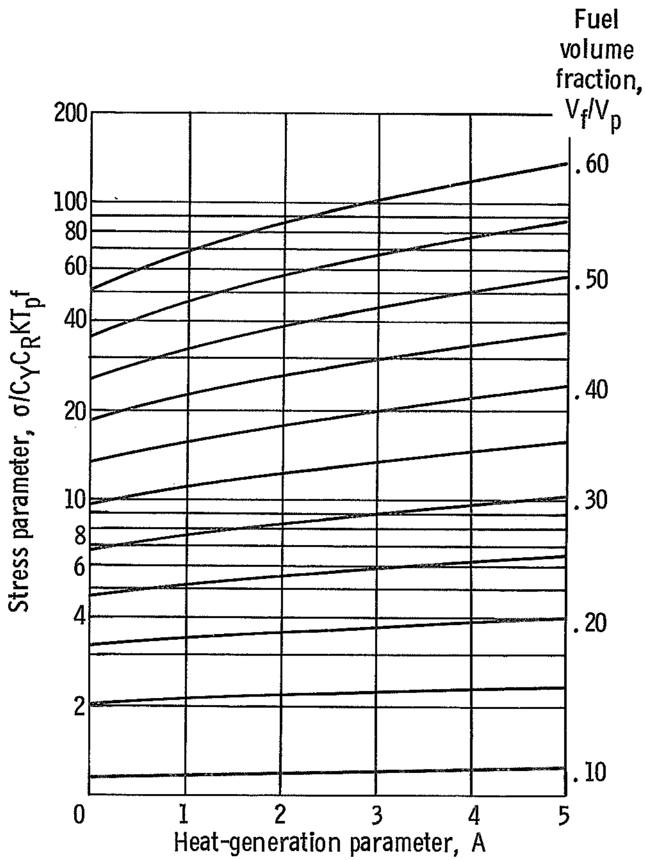


Figure 6. - Design curve of figure 5. Linear interpolation may be used to determine stress parameter.

and 6. The stress parameter for values of n other than 2 may be obtained from figures 5 or 6 and equation (4). Figures 5 and 6 and equation (4) summarize this stress analysis.

The maximum permissible value for the stress parameter is dependent upon the maximum allowable stress. The design of a fuel pin to operate for a long period of time at a high surface temperature must consider the effect of creep. The amount of creep permitted is limited for two reasons. First, it must, of course, be less than that corresponding to rupture; and second, it must not be so great as to significantly affect reactor operation by restricting coolant flow. The limiting stress value also must take into account any variation from nominal design conditions. The creep rate of the clad material may be expressed as a function of n. The exponent n will lie between 4 and 6 for most refractory materials; therefore, the creep rate is extremely sensitive to stress level. Because of this sensitivity, most of the fuel element creep will occur near the end of reactor life when the stress is a maximum.

ILLUSTRATIVE EXAMPLE

An illustrative example is presented to show the use of the graphs of the report for determining a minimum reactor size based on a maximum allowable creep. A list of design conditions is given, including a hypothetical reactor criticality curve and a creep rate equation for the clad material. The reactor criticality curve is representative of a fast spectrum reactor using UO_2 as the fuel. The creep property of the clad material is similar to but not equal to the creep property of tungsten. The values presented in this example are intended only to illustrate the use of the analysis.

Reactor design conditions:

(1) Reactor power, Q_R, MW 2.5
(2) Operating time, τ, sec (hr) . . . . 1.08×10^8 (30 000)
(3) Fuel pin clad temperature, T_p, $^{\circ}$R (K) 2400 (1333)
(4) Fuel pin diameter, D_p, in. (mm) 0.750 (19.1)
(5) Maximum allowable creep, ϵ, percent 1
(6) Reactor criticality . . . . Fig. 7
(7) Reactor length-to-diameter ratio 1
(8) Total fuel pin volume/Reactor volume, V_{p_R}/V_R 0.80
(9) Radial peak-to-average power density ratio 1.20
(10) Variation of V_f/V_p throughout reactor . . . . Uniform

Constants:

(11) Fission gas yield, C_Y 0.125
(12) Fission gas released, C_R 1.0
(13) Fuel thermal conductivity, k, Btu/(hr)(ft)($^{\circ}$R) (J/(m)(sec)(K)) 1.70 (2.94)
(14) Heat energy released per fission, E, Btu/fission
(W-sec/fission) . . . . 2.62×10^{-14} (2.75×10^{-11})

Clad creep properties:

(15) Tangential creep rate, $\dot{\epsilon}$, sec^{-1} (for biaxial stress, ref. 4) . . . $(3/4)^{(n+1)/2} a\sigma^n$
(16) Exponent on stress, n 6
(17) Coefficient, a . . . . Ge^{-H/T_p}
(18) Value for G, sec^{-1} psi^{-6} (sec^{-1}(N/cm^2)$^{-6}$) . . . . 3.55×10^{-14} (5.15×10^{-14})
(19) Value for H, $^{\circ}$R (K) . . . . 10^5 (5.56×10^4)

It is desired to determine the minimum reactor size consistent with these design conditions and the values of the reactor variables which correspond to this minimum size.

First, the end-of-life stress for a range of reactor diameters will be calculated. The average value of the fission density f for any reactor size may be obtained from equation (11)

$$f_{ave} = \frac{q_{ave}\tau}{E} = \frac{\left(\frac{Q_R}{V_R}\right)\tau}{\left(\frac{V_{f_R}}{V_R}\right)E}$$

or, using the radial peak-to-average power density ratio (condition 9) to obtain the average fission density in the highest power fuel pin

$$f = \frac{1.2\left(\frac{Q_R}{V_R}\right)\tau}{\left(\frac{V_{f_R}}{V_R}\right)E}$$

A fuel volume fraction for the reactor V_{f_R}/V_R is selected and the corresponding reactor diameter is read from the criticality curve of figure 7.

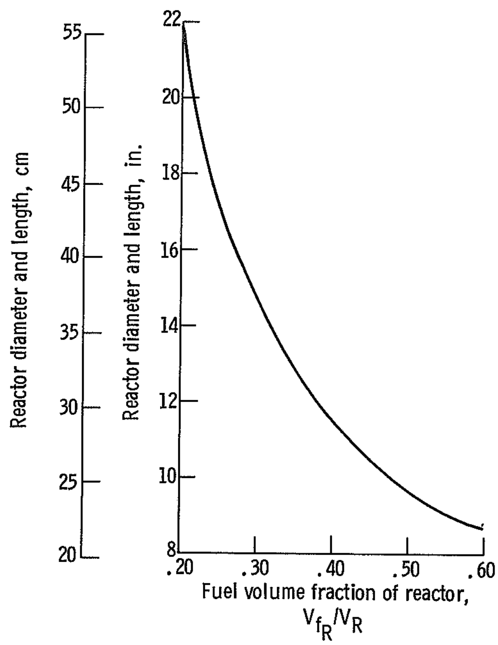


Figure 7. - Hypothetical reactor criticality curve.

For example, at $V_{f_R}/V_R = 0.36$,

$D_R = 12.6$ in. (32.0 cm) (fig. 7)

$V_R = \frac{\pi D_R^3}{4} = 1568$ in.^{3} (25 700 cc) (condition 7)

$\frac{Q_R}{V_R} = 1595$ w/m^3 (97 w/cc) (condition 1)

$f = 2.11\times10^{22}$ fissions/in.^{3} (1.29×10^{21} fissions/cc) (eq. (1); conditions 1, 2, and 14)

$A = 1.89$ (eq. (13); conditions 2, 3, 4, 13, and 14)

$\frac{V_f}{V_p} = \frac{V_{f_R}}{V_R} \times \frac{V_R}{V_{p_R}} = 0.45$ (condition 8)

$\frac{V_v}{V_p} = 0.24$ (fig. 2)

$B_{n=2} = 25.3$ (fig. 6)

$B_{n=6} = 28.0$ (eqs. (4) and (8))

$\sigma = 11\ 900$ psi (8200 N/cm^2) (eq. (8); conditions 3, 11, and 12)

Repeating the calculation for other values of V_{f_R}/V_R, a variation of stress with reactor diameter is obtained as shown in figure 8.

The maximum allowable stress corresponding to condition 5 is then calculated from the creep rate equation (condition 15). Assuming a linear variation of stress with time, the maximum allowable stress for a 1-percent creep in 30 000 hours is 6300 psi (4350 N/cm^2).

The reactor diameter associated with a fuel pin stress of 6300 psi (4350 N/cm^2) is the limiting minimum diameter. If a smaller reactor diameter were used the fuel pin creep would be greater than 1 percent. The reactor and fuel pin characteristics that will satisfy the design conditions are:

Minimum reactor diameter D_R, 14.0 in. (35.6 cm) (fig. 8)

$$\frac{V_{f_R}}{V_R} = 0.32 \qquad \text{(fig. 7)}$$

$$\frac{V_f}{V_p} = 0.40 \qquad \text{(condition 8)}$$

$$\frac{V_v}{V_p} = 0.25 \qquad \text{(fig. 2)}$$

$$\frac{V_c}{V_p} = 0.35 \qquad \text{(fig. 2)}$$

$t = 0.071$ in. (1.81 mm) (fig. 2)

$$\frac{t_f}{t} = 1.60 \qquad \text{(fig. 2)}$$

$A = 1.54$ (eq. (13))

$$\frac{T_g}{T_p} = 1.25 \qquad \text{(fig. 4)}$$

$T_g = 3000^\circ$ R (1667 K)

Although the case presented is hypothetical, a curve of stress and reactor diameter for an actual case would be similar to figure 8. A change of 1 inch (2.54 cm) in reactor diameter could have a marked change in the stress of the fuel pin clad. In the example, a 1-inch (2.54-cm) increase in reactor diameter, 14.0 to 15.0 inches (35.6 to 38.1 cm) reduced the stress by one-third, from 6300 psi (4350 N/cm^2) to 4200 psi (2900 N/cm^2).

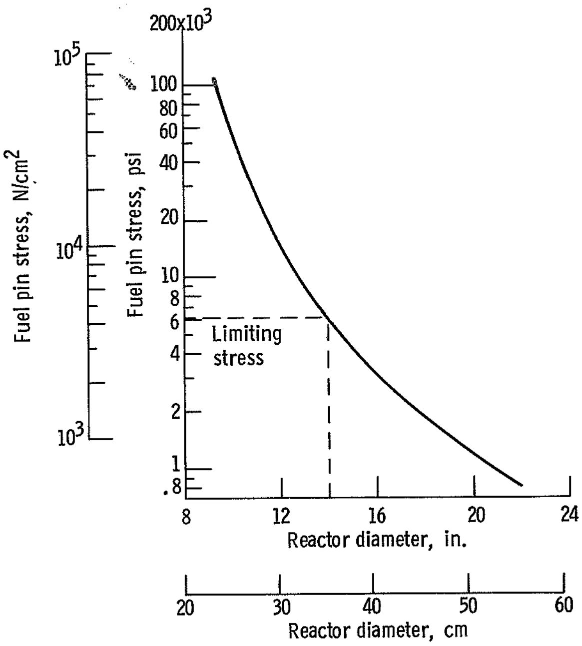


Figure 8. - Variation of fuel pin stress with respect to reactor diameter for illustrative example. Minimum reactor diameter is indicated for limiting stress that will yield a 1 percent creep of fuel pin.

CONCLUDING REMARKS

An analytical investigation is presented of a cylindrical fuel pin for the stress in the clad due to the accumulation of gaseous fission products within a central void. The case considered is for negligible stress imposed by fuel swelling, that is, the fuel strength is low relative to the clad strength or most of the fission products are released from the fuel. The fuel pin consists of three concentric volumes, a central void, the fuel, and the outer clad. The clad serves as a protective cover for the fuel and as a containment vessel for the gaseous fission products. The stress in the clad is the calculated stress for an internally pressurized thick-walled cylinder under steady-state creep.

The analysis shows that an optimum combination of the void and clad volumes exists for each value of fuel-to-pin volume ratio. This optimum yields a minimum stress in the clad.

The determination of void and clad volumes for a prescribed fuel-to-pin volume ratio is simplified by calculating the optimum void-clad combination as determined from the thin-walled hoop stress equation and with no temperature drop across the fuel. The stress calculated with this void-clad combination is slightly greater than the minimum

stress resulting from optimum void-clad combination.

Equations and graphs are presented for the calculation of the stress in the clad. A stress parameter is shown as a function of a heat generation parameter and the fuel-to-pin volume ratio.

An illustrative example is presented showing the use of the graphs and equations of this report in conjunction with a hypothetical reactor criticality curve and a creep rate equation for the fuel pin clad material. A minimum reactor diameter was calculated based on the stress required to give a 1-percent creep in the fuel pin.

Lewis Research Center,
National Aeronautics and Space Administration,
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APPENDIX - SYMBOLS

All symbols are to be used with consistent units. Symbols used in the example may have specific units and are indicated with the example.

A	heat generation parameter, $EfD_p^2/16k\tau T_p$, nondimensional
a	coefficient for creep rate equation
B	stress parameter, $\sigma/C_Y C_R KfT_p$, nondimensional
C_R	fraction of fission gases released
C_Y	fraction of fission products that are gaseous
C_1, C_2	constants of integration (eq. (8))
D_p	diameter of fuel pin
D_R	diameter of reactor
E	heat energy released per fission
f	number of fissions per unit volume of fuel
G, H	constants for coefficient of creep rate equation shown in example
K	Boltzmann constant
k	thermal conductivity of fuel
N	number of molecules
N_F	total number of fissions
n	exponent on stress in the creep equation
p	gas pressure
Q_R	reactor power
q	heat generation rate per unit volume of fuel
r	radius
r_p	outside radius of fuel pin
r_1	inside radius of fuel material
r_2	outside radius of fuel material
T	temperature
T_g	gas temperature
T_p	fuel pin clad temperature
t	clad thickness
t_f	fuel thickness
V_c	clad volume
V_f	fuel volume
V_{f_R}	reactor fuel volume
V_p	pin volume
V_{p_R}	reactor pin volume
V_R	reactor volume
V_v	void volume
ϵ	creep
$\dot{\epsilon}$	tangential creep rate
σ	tangential stress
τ	time

REFERENCES

1. Blake, L. R.: Achieving High Burn-Up in Fast Reactors. J. Nucl. Energy, Pts. A & B, vol. 14, Apr. 1961, pp. 31-48.
2. Miller, John V.: Characteristics of a Space-Power Nuclear Reactor with Considerations for Venting or Containing Gaseous Fission Products. NASA TM X-1631, 1968.
3. Rom, Frank E.; Finnegan, Patrick M.; and McDonald, Glen E.: Very High Burnup Vapor Transport Fuel Pin Concept for Long Life Nuclear Reactors. NASA TN D-4860, 1968.
4. Finnie, Iain; and Heller, William J.: Creep of Engineering Materials. McGraw-Hill Book Co., Inc., 1959, p. 184.
5. Glasstone, Samuel: Principles of Nuclear Reactor Engineering. D. Van Nostrand Co., Inc., 1955, pp. 661-662.

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